Homework 5 - due Monday, August 7 at 10:00 AM

A real **vector space** is a nonempty set V of objects, called *vectors*, on which are defined two operations, called *addition* and *multiplication by scalars*(real numbers), subject to the ten axioms (or rules) listed below. The axioms must hold for all vectors **u**, **v**, and **w** in V and for all scalars c and d.

1. The sum of **u** and **v**, denoted by **u+v**, is in V.
2. **u+v=v+u**.
3. **(u+v)+w=u+(v+w)**.
4. There is a **zero** vector **0** in V such that **u+0=u**.
5. For each **u** in V, there is a vector **-u** in V such that **u+(-u)=0**.
6. The scalar multiple of **u** by c, denoted c**u**, is in V.
7. c(**u+v**)=c**u**+c**v**.
8. (c+d)**u**=c**u**+d**u**.
9. c(d**u**)=(cd)**u**.
10. 1**u**=**u**.

Vector Spaces and Linear Transformations Practice

Make sure to justify your solution for each problem.

1. Let W be the set of points on the line $y = 2x + 1$. That is, let

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = 2x + 1 \right\}.$$

   Is W a vector space? Prove or disprove.

2. Prove that the set of all rational numbers is NOT a vector space over $\mathbb{R}$.

3. Prove that the map $T : \mathbb{R}^2 \to \mathbb{R}^3$ that maps a vector $\mathbf{v}$ to the vector $\mathbf{0}$ is a linear transformation.

4. Suppose that T is the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^5$ that maps a vector

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \text{ to } \begin{bmatrix} -v_1 - 7v_3 \\ v_2 \\ v_1 \\ 0 \\ v_2 - v_3 \end{bmatrix}.$$

   Find a matrix A such that $T(\mathbf{v}) = A\mathbf{v}$.

5. Is the vector $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ in $\text{Span}\left\{ \begin{bmatrix} -4 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$?

6. Let $\mathbf{A} = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$.

(a) Determine if $\mathbf{u}$ is in $NulA$.

(b) Could $\mathbf{u}$ be in $ColA$?